

corrosion at the tips of microcracks in the specimen. This causes the cracks to lengthen. After a period of time under a sustained tension, a crack reaches a critical length [Jaeger, 1962, p. 85] and propagates unstably, causing fracture of the specimen.

Charles [1958] considered a highly elliptical hole of major axis  $L$  in a flat glass plate subject to an average tensile-stress  $S_y$  in a direction perpendicular to the major axis of the crack. He suggested

$$\dot{L}_x = f(S_x) + k \quad (19)$$

where  $\dot{L}_x$  is the velocity of the crack in the  $x$  direction.  $S_x$  is the tensile stress at the tip of the crack tangential to the crack surface;  $k$  is the corrosion rate at zero tangential stress. Suppose

$$f(S_x) = c(S_x/S_{cr})^n \quad (20)$$

where  $n$  is a positive constant,  $c$  is the maximum velocity of the crack, and  $S_{cr}$  is the tensile strength of the atomic bonds at the crack tip. As

$$S_x/S_y = 2(L/r)^{1/2} \quad (21)$$

$$S_{cr}/S_y = 2(L_{cr}/r)^{1/2} \quad (22)$$

where  $r$  is the curvature at the crack tip,  $L_{cr}$  is the length at which occurs the critical stress  $S_{cr}$  for rupture of bonds at the crack tip. Equations 21 and 22 are derived from the theory of stress concentrations around holes in perfectly elastic bodies [Jaeger, 1962, p. 85].

Substituting equations 20 and 21 in equation 19 gives equation 23,

$$\dot{L}_x = c(L/L_{cr})^{n/2} + k \quad (23)$$

Charles suggested that, if static fatigue is to take place, stress-activated corrosion must occur at a much greater rate than stress-free corrosion. Hence the crack would grow with constant curvature  $r$  until it reaches its critical length  $L_{cr}$ . If stress-free corrosion were as important as stress-activated corrosion, the crack growth would occur with increasing radius of curvature, and the stress concentration might be seriously reduced.

The corrosion rate at zero stress  $k$  can therefore be neglected in equation 23 by comparison with the stress-dependent corrosion rate.

The temperature dependence of crack growth can be introduced by the assumption that corrosion is a rate process with an activation energy

$A$ . The experimentally determined activation energy of the process below 150°C is close to that for the diffusion of sodium atoms in glass. Charles suggested that the sodium atoms catalyze the hydrolysis of the oxygen-silicon bond in glass by creating free hydroxyl ions. Equation 23 can be written

$$\dot{L}_x = B(L/L_{cr})^{n/2} \exp(-A/KT) \quad (24)$$

Integrating equation 24 with respect to time gave equation 25,

$$\begin{aligned} \int_{L_0}^{L_{cr}} dL(L/L_{cr})^{-n/2} \\ = \int_0^{t_f} B \exp(-A/KT) dt \\ [2L_{cr}/(n-2)][(L_{cr}/L_0)^{(n-2)/2} - 1] \\ = B \exp(-A/KT)t_f \end{aligned} \quad (25)$$

When  $n$  and  $t_f$  are large, equation 25 can be written,

$$\begin{aligned} [2L_{cr}/B(n-2)] \\ \cdot \exp(A/KT)(L_{cr}/L_0)^{(n-2)/2} = t_f \end{aligned} \quad (26)$$

Taking logarithms of equation 26,

$$\log t_f = (n/2) \log L_{cr} - \log D \quad (27)$$

where

$$D = L_0^{(n-2)/2} (B/2)(n-2) \exp(A/KT)$$

Equation 22 can be rewritten as equation 28,

$$L_{cr} = rS_{cr}^2/4S_y^2 \quad (28)$$

Substituting equation 28 in equation 27,

$$\log t_f = -n \log S_y - \log D' \quad (29)$$

where

$$D' = (rS_{cr}^2/4)^{-n/2} \cdot D$$

Equation 29 gave the static-fatigue law (equation 18). The parameter  $n$  can be determined from the slope of a  $\log t_f - \log S_y$  plot. Charles [1958] reported a value of about 16.

The growth of subcritical cracks under tension has now been directly observed in glass microscope slides [Wiederhorn, 1967] and in sapphire [Wiederhorn, 1968].

It was found that the growth of a crack can be divided into two stages: a stage where crack